

MATH 2D Prep: Integration Techniques

1. Evaluate $\int_1^2 xe^{x^2} dx$

Solution:

$$\begin{aligned}\int_1^2 xe^{x^2} dx &= \int_{x=1}^2 \frac{1}{2} e^{x^2} d(x^2) = \int_{x=1}^2 \frac{1}{2} e^u du && (u = x^2, \quad du = 2x dx, \quad x dx = \frac{du}{2}) \\ &= \int_{u=1}^4 \frac{1}{2} e^u du && (x = 1, \quad u = 1; \quad x = 2, \quad u = 4) \\ &= \left. \frac{1}{2} e^u \right]_1^4 = \frac{e^4 - e}{2}\end{aligned}$$

2. Evaluate $\int_0^\pi x^2 \cos(x) dx$

Solution:

$$\begin{aligned}\int_0^\pi x^2 \cos(x) dx &= \int_0^\pi x^2 d(\sin(x)) = x^2 \sin(x) \Big|_0^\pi - \int_0^\pi \sin(x) 2x dx \\ &= 0 - 2 \int_0^\pi x \sin(x) dx \\ &= 2 \int_0^\pi x d(\cos(x)) \\ &= 2 \left\{ [x \cos(x)]_0^\pi - \int_0^\pi \cos(x) dx \right\} \\ &= 2 \{ [-\pi - 0] - [\sin(x)]_0^\pi \} = -2\pi\end{aligned}$$